Backwater number scaling of alluvial bed forms

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Abstract The backwater number, $B_w$, compares the backwater length scale to the length scale of alluvial bed forms. We derive theory to show that $B_w$ plays an important role in determining the behavior and scaling of morphodynamic systems. When $B_w \gg 1$, spatial patterns in deposition and erosion derive from flow accelerations associated with changes in flow depth, and bed evolution is akin to a kinematic wave. When $B_w \ll 1$, the spatial pattern of shear stress is determined by variations in energy slope, and alluvial beds experience topographic dispersion. This theory is confirmed using a numerical model and data compiled from the literature. We present a data set of $B_w$ for bed forms ranging from dunes to river deltas, including field and experimental measurements. For field-scale measurements, we find that dunes have $B_w > 49$, braid bars exist in the range $B_w = [7.1, 17]$, meanders have a range $B_w = [7.1, 18]$, and river mouth deposition ranges over $B_w = [7.4, 29]$. Further, alluvial morphologies that are easily recreated in the laboratory (dunes and avulsions) have overlapping field and laboratory $B_w$ ranges. In contrast, alluvial forms that have traditionally been difficult to recreate (meanders and river mouth processes) have field $B_w$ that are difficult to match in laboratory settings. Large experimental Froude numbers are shown to reduce experimental $B_w$ and incite diffusional behavior. Finally, we demonstrate the utility of $B_w$ scaling for estimating fundamental scales in sedimentary systems.

1. Introduction

The transport of sediment across planetary surfaces produces a panoply of regular alluvial morphologies at scales from centimeters to hundreds of kilometers that evolve with a variety of behaviors. The hydrologic and sedimentologic conditions that control the behaviors and length scales (or wavelengths) of these forms have been studied extensively for dunes [Smith, 1970; Julien and Klaassen, 1995], bars [Ikeda, 1984; Hundey and Ashmore, 2009], point bars and associated meander bends [Ikeda et al., 1981; Seminara, 2006], river mouth processes [Edmonds and Slingerland, 2007; Hoyal and Sheets, 2009; Rowland et al., 2009], and river deltas [Jerolmack and Swenson, 2007]. The evolution of nonequilibrium pulses and waves of sediment has also been closely investigated [Lisle et al., 2001; James, 2006; East et al., 2015].

Any deviation from a straight, prismatic, uniformly sloping channel can be called a bed form, from ripples to river deltas. Equilibrium bed forms are morphologic deviations that exist near dynamic equilibrium with flow and sediment transport fields. River dunes under steady flow conditions are a classic example of an equilibrium bed form type because they statistically maintain their geometry as they migrate downstream. Dispersive bed forms, on the other hand, do not persist in equilibrium with their environment. Sediment pulses delivered to mountain rivers are dispersive under certain conditions. Over time, the bed deviation associated with the pulse can be smeared away. Understanding the patterns of evolution for both equilibrium and dispersive bed forms is important for stream restoration, river navigation, and stratigraphic interpretation.

Despite the fact that bed forms of all types arise and evolve from sediment transport, no framework exists for universally comparing bed forms to one another, either between the same type of bed form in different systems or between different bed form types in the same system. Such a framework would be valuable because bed forms do not exist independently from one another. Examples include sand bars that are built from dunes and coexist in the same river reach [Mohrig and Smith, 1996; Ashworth et al., 2000], braid bars that form on dispersive sediment pulses [Lisle et al., 1997; East et al., 2015], and river deltas that are constructed from smaller delta lobes or mouth bars [Frazier, 1967; Edmonds and Slingerland, 2007]. A framework for generically comparing many bed form types could provide perspective on different patterns of evolution for colocated bed forms at disparate scales.
A universal framework for investigating alluvial bed forms would ideally explain the evolution of all types of bed forms based on a few key aspects of the system that could be quantified by dimensionless numbers. In an effort toward this goal, we investigate a variety of bed form types by relating their length scales to the hydrodynamic environment in which they evolve through the backwater number, originally defined by Paola and Mohrig [1996]:

\[ Bw \equiv \frac{L_{bw}}{L} = \frac{H_0}{S_0L} \left(1 - Fr_0^2 \right), \]  

where \( L_{bw} \) is the backwater length scale \( H_0/S_0 \left(1 - Fr_0^2 \right) \) normalized by the length scale \( L \) of a given bed form, be it a dune, river delta, or sediment pulse. \( H_0 \) (m) is the characteristic flow depth, \( S_0 \) (–) is the characteristic energy slope gravity, and \( Fr_0 \) is the characteristic Froude number \( (Fr_0 = U_0/\sqrt{gH_0}) \), where \( U_0 \) is the characteristic downstream velocity (m/s) and \( g \) is acceleration due to gravity, m/s². \( L_{bw} \) is often used to describe hydrodynamic environments [Chaudhry, 2007; Jerolmack and Swenson, 2007]. The Froude number term \( (1 - Fr_0^2) \) is sometimes neglected from \( L_{bw} \) by employing an assumption that flow is significantly subcritical \((Fr_0\ll1)\), but a Froude number near one can significantly reduce \( L_{bw} \) in some natural and experimental settings. \( L_{bw} \) scales the distance over which flow in a prismatic channel of constant slope adjusts to a morphologic perturbation such as a bed slope change (e.g., waterfall or river impoundment) or a perturbation imposed by a bed form [Chaudhry, 2007]. Therefore, when \( Bw \ll 1 \), flow can fully adjust to the perturbation imposed by a bed form. In contrast, when \( Bw \gg 1 \), the flow does not fully adjust over the length scale of the bed form of interest [Paola and Mohrig, 1996; Chaudhry, 2007]. Hence, for a hydrodynamic environment scaled by \( L_{bw} \) and a bed form scaled by \( L \), a bed form can experience different hydrodynamic forcing dependent upon \( Bw \).

The backwater number concept has proven useful for studying river deltas. Jerolmack and Swenson [2007] showed that the distance from delta apex to the shoreline of many deltas is well approximated by \( L_{bw} \), suggesting that \( Bw \approx 1 \) is a rule of thumb for the avulsions that distribute sediment across the delta. Subsequent studies have invoked backwater effects on flow within \( L_{bw} \) of coastlines to explain the aggradation that leads to avulsions at a delta apex, erosion of a channel bed near a river mouth, and basinward migration of a delta avulsion node during delta progradation [Chatantanavet et al., 2012; Lamb et al., 2012; Nittrouer et al., 2012; Ganti et al., 2014]. However, backwater effects are not limited to within \( L_{bw} \) of the shoreline. Indeed, all bed forms evolve in relation to sediment transport and fluid stress fields with potential contributions from backwater effects whether they are located in mountain catchments, midcontinental environments, or near a coastline.

Laboratory study of alluvial bed forms requires some form of scaling to be related to field-scale systems (see Kleinhans et al. [2014] for a comprehensive review), yet scaling via \( Bw \) has not been used in this context. Dynamical scaling, which relates the magnitudes of various forces imparted by fluid flow, has proven difficult in experimental settings. Nevertheless, many sedimentological experiments have been labeled unreasonably effective at reproducing behavior and stratigraphy found at the field scale [Paola et al., 2009]. In particular, Paola et al. [2009] note the behavior of dunes, braided rivers, and avulsive river deltas are reproduced faithfully. Unfortunately, reproduction of bed forms at laboratory scales is not universally simple. With notable exceptions, the reproduction of river meandering and channel mouth processes in the laboratory remains difficult compared to their frequent formation on the Earth’s surface. In the case of river meandering, sinuous single thread channels have been created for hundreds of hours [Braudrick et al., 2009; Van Dijk et al., 2012], but channel sinuosities remain small compared to field-scale systems or require sediment cohesion or an oscillating upstream boundary condition. There are also disparities in scaling channel mouth processes; field-scale delta channels have been found to be erosional in their downstreammost reaches [Nittrouer et al., 2011b; Shaw et al., 2013; Shaw and Mohrig, 2014]. However, channel mouths in laboratory settings are primarily depositional [Hoyal and Sheets, 2009; Van Dijk et al., 2009; Reitz et al., 2010; Reitz and Jerolmack, 2012]. The mechanistic differences between bed forms that are easy and difficult to recreate in laboratory settings remain unclear.

We posit that \( Bw \) plays an unrecognized role in controlling these systems that has not been appreciated in field settings or laboratory models.

To explore controls on the scaling of sizes and behaviors of bed forms, we examine the potential role of backwater effects on interactions between bed topography and flow and transport fields. We begin by performing a scaling analysis of an analytical model of bed evolution (section 2) and use it as a basis for exploring three
hypotheses: (1) $B_w$ scaling controls the continuum of bed form motion from translation to dispersion, (2) bed form types have characteristic $B_w$ scales, and (3) bed forms that are easily produced in laboratory settings have $B_w$ scales that closely match nature. Second, we demonstrate that the results of a numerical model of topographic evolution are consistent with hypothesis 1 and compare predicted and observed behaviors of bed forms from observations existing in the literature. Third, we compile a data set of more broadly defined alluvial bed forms from field and laboratory settings to test hypotheses 2 and 3. Finally, we demonstrate the utility of $B_w$ scaling by applying it to paleohydraulic reconstruction problems on Earth and Mars.

2. Theory of Backwater Scaling

We aim to develop a theory for the behavior of alluvial beds by deriving a backwater number-dependent expression for one-dimensional evolution in alluvial channels. This will be done by relating the evolution of an alluvial bed to its hydrodynamic controls using a scaling analysis. Although two-dimensional bed forms cannot be fully resolved in a one-dimensional treatment, their stability and migration have also been shown to depend explicitly on accelerations of flow in the downstream direction [Nelson and Smith, 1989; Bridge and Gabel, 1992; Lisle et al., 1997; Ashworth et al., 2000; Edmonds and Singerland, 2007]. Hence, we develop theory for the influence of the backwater number in one dimension and use it as a guide to interpret two-dimensional bed forms.

We begin with sediment mass conservation; temporal changes in the topography of a sediment surface are related to divergence of a sediment transport field through the Exner equation:

$$\frac{\partial \eta}{\partial t} = -\frac{1}{\epsilon} \frac{\partial q_s}{\partial x}. \quad (2)$$

Here $\eta$ is bed elevation (m), $t$ is time (s), $\epsilon$ is volume concentration of sediment in a bed ($-$), and $q_s$ is sediment flux (m$^2$/s) in the flow-parallel ($x$) direction (Figure 1). The Exner equation can be formulated to address a wide range of processes such as long-term morphologic change by including subsidence or short-term change by including temporal variability in suspended sediment concentration [Paola and Voller, 2005]. In general, bed forms are the result of morphodynamic interactions of topography, fluid stress, and sediment flux and are not related to subsidence or time-variable sediment concentrations. For this reason those parameterizations are neglected here.

The next step is to couple equation (2) to a model for sediment flux ($q_s$). Typically, sediment flux is treated as a power law function of shear stress [e.g., Meyer-Peter and Muller, 1948; Jerolmack and Mohrig, 2005; García, 2007]:

$$q_s = \alpha \left( \tau_b - \tau_c \right)^\beta, \quad (3)$$

where $\tau_b$ is the boundary shear stress applied by a flowing fluid to a granular bed (kg/m/s$^2$), $\tau_c$ is the critical shear stress required for sediment motion (kg/m/s$^2$), $\alpha$ is a coefficient that incorporates particle size, density, gravity, and bed friction, and $\beta$ is an exponent generally between 3/2 and 5/2 largely depending on the modes.
of sediment transport included [García, 2007]. By holding $\alpha$ and $\beta$ constant and setting $\tau_c \approx 0$, we ignore the effects associated with the threshold of sediment motion, grain mixtures, or sources of resistance to flow other than sediment grains. Because of these omissions, the model derived here is valid for scaling but should not be applied to generate exact predictions of sediment flux or rates of topographic change under backwater influence.

Combining (2) and (3) relates bed evolution to spatial gradients in boundary shear stress:

$$\frac{\partial \eta}{\partial t} = \frac{\alpha \beta \gamma}{\epsilon} \frac{\tau_b^{\beta-1}}{\tau_b} \frac{\partial \tau_b}{\partial x}. \tag{4}$$

The next step is to include a model for boundary shear stress in order to close this problem and scale morphologic change with backwater effects. An appropriate model for boundary shear stress comes from depth-averaged, hydrostatic flow over an alluvial surface given by the St. Venant equations. These equations allow for a spatially varying depth-averaged flow field. The momentum balance can be written as

$$\frac{\partial U}{\partial t} + U \frac{\partial U}{\partial x} = -g \frac{\partial \eta}{\partial x} - g \frac{\partial H}{\partial x} - \frac{\tau_b}{\rho}. \tag{5}$$

where $U$ is the depth-averaged flow velocity (m/s) in the downstream ($x$) direction, $g$ is acceleration due to gravity (m/s²), and $\rho$ is fluid density (kg/m³). Bed forms can form, translate, and disperse in steady flow, so we assume that $\frac{\partial U}{\partial t} = 0$. Under the assumption of incompressible, quasi-steady, one-dimensional flow, conservation of fluid mass is given by

$$\frac{\partial (UH)}{\partial x} = 0. \tag{6}$$

Expanding this yields

$$\frac{\partial U}{\partial x} = -\frac{U}{H} \frac{\partial H}{\partial x}. \tag{7}$$

Substituting this into equation (5) with the assumptions described above produces

$$\tau_b = \rho g H S_f, \tag{8}$$

with $S_f$, the energy slope (unitless), given by

$$S_f \equiv -\frac{\partial \eta}{\partial x} - \frac{\partial H}{\partial x} (1 - Fr^2). \tag{9}$$

$S_f$ is defined to be positive for unidirectional flow when the water surface elevation decreases with increasing $x$. Because we have a model for topographic change (4) that depends on spatial gradients in boundary stress, equation (8) is differentiated with respect to $x$. Substituting the result into equation (4) produces

$$\frac{\partial \eta}{\partial t} = \frac{\alpha \beta \rho g}{\epsilon} \frac{\tau_b^{\beta-1}}{\tau_b} \left( \frac{\partial H}{\partial x} S_f + H \frac{\partial S_f}{\partial x} \right). \tag{10}$$

Given that $\frac{\alpha \beta \rho g}{\epsilon} \tau_b^{\beta-1}$ is always positive in depth-averaged flow, equation (10) shows that the sign of $\frac{\partial \eta}{\partial t}$, and thus local erosion and deposition patterns, is determined by the relative magnitudes of spatial gradients in water depth ($\frac{\partial H}{\partial x}$) and energy slope ($\frac{\partial S_f}{\partial x}$). We explore their relative magnitudes by rearranging equation (9) to produce

$$1 + \frac{1}{S_f} \frac{\partial H}{\partial x} (1 - Fr^2) = -\frac{1}{S_f} \frac{\partial \eta}{\partial x}. \tag{11}$$

Equation (11) relates local spatial changes in flow depth directly to the energy slope on the left-hand side and the bed slope to the energy slope on the right-hand side. We note that equation (11) does not explicitly involve spatial changes in energy slope ($\frac{\partial S_f}{\partial x}$). However, a scaling analysis of (11) can be used to determine controls on equation (10). For characteristic values of flow depth ($H_0$), energy slope ($S_{f0}$), and Froude number ($Fr_0$) averaged over the length of the bed form ($L$), equation (11) together with (1) produces

$$1 + Bw \sim \left| -\frac{1}{S_f} \frac{\partial \eta}{\partial x} \right|. \tag{12}$$
Equation (12) shows that $1 + Bw$ scales the variation in the ratio of local bed slope to local energy slope over a bed form. The scale on the left-hand side of (12) should not be confused as a characteristic bed slope divided by $S_f$. Figure 1 shows that $S_f$ can be parallel to the characteristic bed slope averaged over $L$ (causing their ratio to be $\approx 1$) regardless of $Bw$. However, when $Bw \ll 1$ (Figure 1a), \[ \left| \frac{1}{Bw} \frac{\partial}{\partial x} \right| \approx 1 \] at all points over the bed form, but when $Bw \gg 1$ (Figure 1b), \[ \left| \frac{1}{Bw} \frac{\partial}{\partial x} \right| \gg 1 \] for almost all locations on the bed form.

The magnitude of $Bw$ influences bed evolution through equations (9), (10), and (12). In an environment where $Bw \ll 1$, $\frac{\partial}{\partial x}$ is approximately negative one times the energy slope; $\frac{\partial}{\partial x} \approx -S_f$ (Figure 1a). This simplification can be substituted into (9) to show that $\frac{\partial}{\partial x} (1 - Fr^2) \approx 0$. If these conditions are the result of minimal changes in water depth ($\frac{\partial}{\partial x} \approx 0$; Figure 1a) and $Fr_0 \ll 1$, then and equation (10) is reduced to

$$\frac{\partial \eta}{\partial t} \sim A_1 \frac{\partial^2 \eta}{\partial x^2}, \tag{13}$$

where $A_1 = \frac{\alpha \beta \gamma}{\epsilon \gamma} \frac{1}{\epsilon} H_0$. This is a diffusion equation akin to that previously produced by Paola et al. [1992] to illustrate diffusive sediment transport over large length scales. We therefore expect that when $Bw \ll 1$ and $Fr \ll 1$, bed topography that deviates from a uniform slope will experience dispersive evolution. Under these conditions, flow depth remains relatively constant, and bed evolution is dictated by parallel fluctuations of the bed and energy slopes discussed above ($\frac{\partial}{\partial x} \approx -S_f$). When $\frac{\partial}{\partial x} (1 - Fr^2) \approx 0$ is brought on by $Fr_0 \approx 1$, the same simplification is not possible because $\partial H/\partial x$ cannot be canceled from equation (10).

In contrast, when $Bw \gg 1$, equation (12) shows that fluctuations in bed slope far exceed fluctuations in the energy slope (Figure 1b). $S_f$ is then negligible in comparison to $\frac{\partial}{\partial x}$ and can be removed from equation (9), producing the approximation $\frac{\partial}{\partial x} \approx \frac{\partial}{\partial x} (1 - Fr^2)$. We are interested in spatial changes to the energy slope, which are expressed by differentiation of equation (9) as

$$\frac{\partial S_f}{\partial x} = -\frac{\partial^2 \eta}{\partial x^2} - \frac{\partial}{\partial x} \left( \frac{\partial H}{\partial x} (1 - Fr^2) \right). \tag{14}$$

However, if $\frac{\partial}{\partial x} (1 - Fr^2)$ is approximately equal to $-\frac{\partial}{\partial x}$, then their derivatives with respect to $x$ will also be approximately equal, making $\partial S_f/\partial x \approx 0$. This result can be used to simplify equation (10) to produce

$$\frac{\partial \eta}{\partial t} \sim -A_2 \frac{\partial \eta}{(1 - Fr_0^2) \partial x}, \tag{15}$$

where $A_2 = \frac{\alpha \beta \gamma}{\epsilon \gamma} \frac{1}{\epsilon} S_f$. Equation (15) shows that when $Bw \gg 1$, the bed evolves as a kinematic wave where the celerity is determined by $A_2$. This theory also reproduces the well-known result that waves translate downstream where $Fr_0$ is less than about 1 and upstream when $Fr_0$ is greater than about 1 [Kennedy, 1963; Engelund and Fredsøe, 1982]. Kinematic behavior occurs when the energy slope is constant and bed evolution is dictated by equal and opposite fluctuations of water depth and bed elevation.

The difference between topographic diffusion when $Bw \ll 1$ and translation for $Bw \gg 1$ shown in this scaling analysis highlights the potential importance of $Bw$ in determining bed evolution patterns. However, several questions remain unanswered. How does evolution change when $Bw$ is small due to $Fr_0$ conditions near 1? Do predictions using $Bw$ capture the evolution of natural bed forms? What are the $Bw$ scales of natural bed forms? These questions are addressed in the remainder of this manuscript.

### 3. Methods

In order to further investigate the control of $Bw$ on bed form scale and behavior, we analyze the results of a simple numerical model based on the scaling analysis in section 2. Then, we perform a literature survey to evaluate the ability for $Bw$ to summarize the behavior and scale of various types of bed forms. Methods for each approach are given below.

#### 3.1. Numerical Model

A numerical model coupling one-dimensional backwater flow to sediment transport and bed evolution is used to test the theoretical predictions made in section 2, specifically exploring the independent influence of $Bw$ and $Fr_0$ on the translation and dispersion of evolving bed forms. The model is basically an implementation...
Table 1. Compiled Data Set of Backwater Scaling of Bed Form Behavior

<table>
<thead>
<tr>
<th>Reference</th>
<th>Bed Form Type</th>
<th>Location/Experiment</th>
<th>$H_b$ (m)</th>
<th>$L$ (m)</th>
<th>$S_i$</th>
<th>$Fr_0$ (-)</th>
<th>$Bw^a$ (-)</th>
<th>Behavior $^b$</th>
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<tr>
<td>Kostaschuk and Villard [1996]</td>
<td>River Dunes</td>
<td>Fraser River, Canada</td>
<td>$1.5 \times 10^0$</td>
<td>$3.3 \times 10^1$</td>
<td>$5.0 \times 10^{-5}$</td>
<td>0.15</td>
<td>$8.9 \times 10^{2}$</td>
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<td>Rhine River, Germany</td>
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<td>Gaeuman and Jacobson [2007]</td>
<td>River Dunes</td>
<td>Missouri River, USA</td>
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<td>$1.1 \times 10^1$</td>
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<td>Mississippi River, USA</td>
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<td>0.11</td>
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<td>North Loup River, USA</td>
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<td>Experiment</td>
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<td>0.35</td>
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<td>River Dunes</td>
<td>Experiment S3</td>
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<td>$4.5 \times 10^{-1}$</td>
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<td>0.23</td>
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<td>Madej and Ozaki [1996]</td>
<td>Sediment Pulse</td>
<td>Redwood Creek, USA</td>
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<td>Meade [1985]</td>
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<td>Sediment Pulse</td>
<td>Experiment 10</td>
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<td>Sklar et al. [2009]</td>
<td>Sediment Pulse</td>
<td>Experiment 21</td>
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<td>Nelson et al. [2015]</td>
<td>Sediment Pulse</td>
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<td>$6.7 \times 10^{-3}$</td>
<td>0.64</td>
<td>$1.7 \times 10^{0}$</td>
<td>M</td>
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</tbody>
</table>

$^a$ $Bw$ calculated using equation (1).

$^b$ Behavior reported in original reference; D = Dispersive, M = Mixed, and T = Translative.

The standard deviation of bed elevation, $\sigma_r$, from which $Bw$ and laboratory experiments. The scientific literature abounds with studies of a wide variety of bed form types from which $Bw$ can be calculated. For each case we calculate $Bw$ from a characteristic water depth ($H_b$), energy...
Table 2. Compiled Data Set for Backwater Scaling of Equilibrium Bed Forms

<table>
<thead>
<tr>
<th>Reference</th>
<th>Bed Form Type</th>
<th>Location/Experiment</th>
<th>$H_0$ (m)</th>
<th>$L$ (m)</th>
<th>$S_{10}$ (--)</th>
<th>$F_{10}$ (--)</th>
<th>$Bw^H$ (--)</th>
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<td>Ashworth et al. [2000]</td>
<td>Braid Bar</td>
<td>Jamuna River, Bangladesh</td>
<td>5.0 × 10^0</td>
<td>4.0 × 10^3</td>
<td>9.0 × 10^-3</td>
<td>0.21</td>
<td>1.3 × 10^1</td>
</tr>
<tr>
<td>Smith [1971]</td>
<td>Braid Bar</td>
<td>Platte River, USA</td>
<td>5.0 × 10^-1</td>
<td>2.8 × 10^1</td>
<td>1.4 × 10^-3</td>
<td>0.06</td>
<td>1.3 × 10^1</td>
</tr>
<tr>
<td>Bridge and Gabel [1992]</td>
<td>Braid Bar</td>
<td>Calamus River, USA</td>
<td>4.0 × 10^-1</td>
<td>3.0 × 10^1</td>
<td>7.5 × 10^-4</td>
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<td>1.7 × 10^1</td>
</tr>
<tr>
<td>Alexander et al. [2013]</td>
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<td>Niobrara River, USA</td>
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<td>Lanzoni [2000]</td>
<td>Braid Bar</td>
<td>Experiment P1801</td>
<td>7.3 × 10^-2</td>
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<tr>
<td>Ashmore [1991]</td>
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<td>1.2 × 10^-2</td>
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<td>1.0 × 10^-2</td>
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</tr>
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<td>Egazi and Ashmore [2009]</td>
<td>Braid Bar</td>
<td>Experiment 8</td>
<td>7.0 × 10^-3</td>
<td>5.9 × 10^-1</td>
<td>1.5 × 10^-2</td>
<td>0.91</td>
<td>1.4 × 10^-3</td>
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<tr>
<td>Gautier et al. [2007]</td>
<td>Meander</td>
<td>Rio Beni, Brazil</td>
<td>1.0 × 10^1</td>
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<td>2.4 × 10^-4</td>
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<tr>
<td>Johannesson and Parker [1985]</td>
<td>Meander</td>
<td>Minnesota River, USA</td>
<td>4.4 × 10^0</td>
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<td>Fisk [1947]</td>
<td>Meander</td>
<td>Mississippi River, USA</td>
<td>1.4 × 10^1</td>
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<td>5.5 × 10^-5</td>
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<td>Andrews [1979]</td>
<td>Meander</td>
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<td>Experiment</td>
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<td>1.6 × 10^0</td>
</tr>
<tr>
<td>Smith [1998]</td>
<td>Meander</td>
<td>Experiment</td>
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<td>Schumm and Khan [1972]</td>
<td>Meander</td>
<td>Experiment</td>
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<td>Van de Lageweg et al. [2013]</td>
<td>Meander</td>
<td>Experiment</td>
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<td>Braudrick et al. [2009]</td>
<td>Meander</td>
<td>Experiment</td>
<td>1.9 × 10^-2</td>
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<td>4.6 × 10^-3</td>
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<td>Nittrouer et al. [2012]</td>
<td>Channel Mouth</td>
<td>Mississippi Birdfoot, USA</td>
<td>2.0 × 10^1</td>
<td>6.0 × 10^4</td>
<td>4.4 × 10^-5</td>
<td>0.14</td>
<td>7.4 × 10^0</td>
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<tr>
<td>Shaw and Mohrig [2014]</td>
<td>Channel Mouth</td>
<td>Wax Lake Delta, USA</td>
<td>1.0 × 10^1</td>
<td>8.0 × 10^3</td>
<td>8.8 × 10^-5</td>
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<td>1.4 × 10^0</td>
</tr>
<tr>
<td>Edmonds and Slingerland [2007, 2008]</td>
<td>Channel Mouth</td>
<td>Mossy Delta Channels, Canada</td>
<td>2.0 × 10^0</td>
<td>1.0 × 10^3</td>
<td>6.7 × 10^-5</td>
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<td>Shaw et al. [2013]</td>
<td>Channel Mouth</td>
<td>Wax Lake Delta Channels, USA</td>
<td>3.0 × 10^-2</td>
<td>5.0 × 10^3</td>
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<td>Rowland et al. [2009]</td>
<td>Channel Mouth</td>
<td>Mississippi River Tie Channel, USA</td>
<td>1.2 × 10^1</td>
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<td>5.2 × 10^-5</td>
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<td>Reitz and Jerolmack [2012]</td>
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<td>Shoreline Perturbation Expt. R20</td>
<td>9.3 × 10^-3</td>
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<td>Powell et al. [2012]</td>
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<td>Expt. MwMs</td>
<td>1.0 × 10^-2</td>
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<td>Hoyal and Sheets [2009]</td>
<td>Channel Mouth</td>
<td>Experiment Agg2</td>
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<td>Edmonds et al. [2009]</td>
<td>Channel Mouth</td>
<td>Expt. DL9</td>
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</tr>
<tr>
<td>Chatanantavet and Lamb [2014]</td>
<td>Channel Mouth</td>
<td>Experiment 2</td>
<td>1.2 × 10^-1</td>
<td>1.0 × 10^0</td>
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<td>Chatanantavet et al. [2012]</td>
<td>Channel Mouth</td>
<td>Parana Delta, Argentina</td>
<td>1.2 × 10^1</td>
<td>2.1 × 10^5</td>
<td>4.0 × 10^-5</td>
<td>0.14</td>
<td>1.4 × 10^0</td>
</tr>
<tr>
<td>Delta Avulsion</td>
<td>Danube Delta, Romania</td>
<td>6.3 × 10^-3</td>
<td>9.5 × 10^4</td>
<td>5.0 × 10^-5</td>
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<td>1.3 × 10^0</td>
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<tr>
<td>Delta Avulsion</td>
<td>Assiniboine River, Canada</td>
<td>4.2 × 10^-3</td>
<td>1.2 × 10^4</td>
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<td>5.3 × 10^-1</td>
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<td>Delta Avulsion</td>
<td>Rhine-Meuse Delta, Netherlands</td>
<td>5.0 × 10^-3</td>
<td>5.1 × 10^4</td>
<td>1.1 × 10^-4</td>
<td>0.23</td>
<td>8.4 × 10^-1</td>
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<td>Delta Avulsion</td>
<td>Orinoco Delta, Venezuela</td>
<td>8.0 × 10^-3</td>
<td>7.8 × 10^4</td>
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<td>0.17</td>
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<td>van Dijk et al. [2009]</td>
<td>Delta Avulsion</td>
<td>Cut and Fill Cycles, Expt. R1</td>
<td>1.0 × 10^-2</td>
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<td>0.32</td>
<td>9.2 × 10^-1</td>
</tr>
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<td>Sittioni et al. [2014]</td>
<td>Delta Avulsion</td>
<td>Experiment</td>
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<td>2.0 × 10^-2</td>
<td>0.58</td>
<td>2.0 × 10^-1</td>
</tr>
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</table>

a Bed form type interpreted from description in the original reference.

b $Bw$ is calculated using equation (1).

slope ($S_{10}$), Froude number ($F_{10}$), and length scale of the bed form of interest ($L$) using equation (1). Care was taken to record characteristic water depths associated with the formative conditions of the bed forms. In studies where there were many bed forms, the mean or median length was used. Where these quantities were not explicitly reported, they were estimated from accompanying data and figures.

Many studies do not explicitly report $S_{10}$, but it can be estimated using one of the following methods. In studies where the spatially averaged water depth does not change over one or several bed forms, $\partial H/\partial x$ ≈ 0 and $S_{10}$ can be approximated by $-\partial H/\partial x$ averaged over the same several bed forms (see equation 9). In studies where water depth changes over the bed form but $Fr^2 < 1$ (many river mouths and delta avulsions), then $S_{10}$ can be approximated by the average water surface slope over the bed form $-\partial (H + \partial H/\partial x)$. In the few cases where neither of these approximations were possible [Edmonds et al., 2009; Reitz and Jerolmack, 2012], then $S_{10}$ was calculated directly using equation (9).
Two data sets were assembled. The first data set was compiled specifically for testing hypothesis 1, which states that the continuum of bed form behavior from translative to dispersive is captured by $B_w$. To do this, we found published works in which a specific behavior was reported for one-dimensional bed forms. This strategy was adopted to limit our possible biases. For partially or completely dispersive behavior this strategy works. That behavior is often reported where observed. For sediment pulses that experienced dispersion or lengthening over time, the initial pulse length is recorded [Cui et al., 2003; Sklar et al., 2009; Nelson et al., 2015]. In contrast, purely translative behavior is the norm for bed forms, and it is therefore not often reported. To overcome this limitation, we argue that equilibrium ripples and dunes unless otherwise stated are examples of translational bed forms [Simons et al., 1965; Allen, 1974; McElroy and Mohrig, 2009]. These data are summarized in Table 1.

A second data set was assembled to test hypothesis 2, which distinct bed form types have characteristic $B_w$ ranges, and hypothesis 3, which bed forms that are easily produced in laboratory settings have $B_w$ scales that closely match nature. We collected published data on braid bars, meanders, channel mouth processes, and delta avulsions and their formative conditions ($H_0$, $S_{fr}$, $L$, and $F_r$). The length scale of bed forms has been defined in various ways by Ferguson [1975] and Williams [1986]. In this study, $L$ is defined as the Cartesian length of a form. Data are summarized in Table 2.

Tests of significance of differences in $B_w$ scaling between field and experimental studies would require many instances of independent measurements for each bed form type in the lab and in the field separately. It is presently not possible to find more than three or four experimental studies on any single type of bed form except for dunes. We used as many experimental studies as we could locate and attempted to roughly match that number of field studies. As a result there is a rather small sample size for each bed form type. We note however that many studies of equilibrium bed forms resolve tens to hundreds of individual bed forms that spontaneously formed characteristic scales. Hence, the uncertainty of each individual point is relatively low, but the uncertainty in the overall distribution of $B_w$ for each type is relatively large. For this reason we limit our statistical discussion to ranges of observed $B_w$, and conclusions reached from their comparison should be tempered by the general paucity of data.

4. Results From Tests of $B_w$ Scaling for Bed Forms

With a theory for backwater number ($B_w$) scaling of bed form evolution established (section 2), we apply it as a basis for exploring our hypotheses: (1) $B_w$ scaling is a predictor of the continuum of topographic behavior for bed forms, from translation to dispersion, (2) $B_w$ scaling of individual bed form types occurs in characteristic ranges, and (3) bed forms that are easily produced in laboratory settings have $B_w$ scales that match nature. First, we demonstrate that the results of a numerical model of topographic evolution are consistent with hypothesis 1 and then compare predicted and observed behaviors of bed forms from observations.
4.1. \( Bw \)-scaled Numerical Model of Bed Evolution

A one-dimensional numerical model further investigates the behavior of bed forms as a function on \( Bw \) and \( F_{0} \). Figure 2 shows the evolution of four sinusoidal bed forms with \( F_{0} = 0.3 \) and \( Bw \) varying between 0.1, 1, and 10. For \( Bw = 10 \) (Figure 2c), the sinusoidal bed forms shift downstream with time and translation dominates. When \( Bw = 0.1 \) (Figure 2a), the initial bed form topography disperses to a uniform bed with almost no translation. At \( Bw = 1 \) both translation and dispersion occur (Figure 2b). Normalized translative and dispersive fluxes vary with \( Bw \) and \( F_{0} \) (Figure 3). In general, normalized translative flux \( (0.5cV_{h_{d}}/\Delta q_{s}) \) increases with increasing \( Bw \) and decreasing \( F_{0} \), although there is a minor decrease around \( Bw \approx 2-3, F_{0} = 0.4-0.6 \) due to non-linearity in the sediment flux \( (\beta = 2.5) \). Importantly, normalized translative flux is largely independent of the characteristic Froude number for \( F_{0} < 0.6 \).

Normalized dispersive flux \( (1 - (0.5cV_{h_{d}}/\Delta q_{s})) \); Figure 3b) is the compliment to normalized translative flux and increases with decreasing \( Bw \) and increasing \( F_{0} \). Translative fluxes are dominant where \( Bw \gg 1 \), and dispersive fluxes are dominant where \( Bw \ll 1 \), although they become equal (for \( F_{0} = 0.3 \)) when \( Bw \approx 0.5 \), rather than at unity. These model results show that \( Bw \) has significant control on advection versus dispersion in a simplified 1-D alluvial system, supporting hypothesis 1. Further, it shows that as \( F_{0} \) approaches 1, the effect is to increase the relative dispersive flux, regardless of \( Bw \).

4.2. Literature Observations of \( Bw \)-Scaled Bed Form Behavior

To relate the theoretical (section 2) and numerical (section 4.1) results to nature, we introduce a literature survey to empirically determine how bed form behavior varies with \( Bw \). Despite the relative ease in calculating \( Bw \), it is not common for studies to document if bed forms evolve through translation or dispersion. Although there is a substantial body of work that aims to characterize sediment pulses, slugs, and supply rate changes in fluvial settings, few of these studies include the necessary information to evaluate backwater scaling. Exceptions are experiments and field studies focused on the controls and implications of dispersion and translation of sediment waves [Madej and Ozaki, 1996; Lisle et al., 1997; Cui et al., 2003; Sklar et al., 2009]. We identified 12 instances where completely or partially dispersive bed behavior is reported (Table 1). Purely dispersive behaviors existing in the literature. Finally, we compile a data set of more broadly defined alluvial bed forms from field and laboratory settings to test hypotheses 2 and 3.
are observed over the range $Bw = [1.9 \times 10^{-2}, 1.6 \times 10^{-2}]$, and mixed behaviors are observed for the range $Bw = [7.5 \times 10^{-2}, 1.3 \times 10^{2}]$. We added to this seven instances of dune evolution that reported enough information to determine $Bw$, which showed a range of $Bw = [4.9 \times 10^1, 2.6 \times 10^4]$.

Within this data set, translatative bed forms are found only at $Bw > 10^1$, dispersive bed forms are found only at $Bw < 10^{-1}$, and mixed behavior was found on the interval $Bw = [10^{-1}, 10^1]$ (Figure 4). This analysis is consistent with the theory described in section 2, with the resulting model (Figure 2), and with hypothesis 1; the magnitude of $Bw$ encapsulates the determinative factors for whether a bed form will evolve through dispersion.

**Figure 5.** Planview sketches of (a) river delta avulsions, (b) river mouth deposition, (c) river meanders, and (d) braid bars. For each panel, gray areas indicate land and white areas indicate water. The avulsion from the right to the left channel in Figure 5a is indicated by an arrow. The characteristic length scale ($L$) is indicated for each bed form. Note that for river meanders (Figure 5c) $L$ is associated with meander wavelength. For braid bars (Figure 5d), $L$ is the mean braid bar length.
or translation. When bed forms are long relative to the backwater length ($L_{bw}$), their topographies are largely dispersive, when they are short relative to the backwater length, they behave as kinematic waves, and when the length scales are approximately equal, mixed dispersive-translative behavior occurs.

4.3. Literature Observations of $Bw$ Scales for Equilibrium Bed Forms

We now turn to our second hypothesis, which the $Bw$ scale of various bed form types clusters in distinct ranges. This is accomplished by evaluating the literature review of $Bw$ for a wide variety of equilibrium bed form types (Tables 1 and 2). Equilibrium dunes from Table 1 are included here, but dispersive bed forms are not included because they do not maintain a length scale during evolution. The compiled data set includes 43 independent observations with sets of delta avulsions (7), channel mouths (12), meanders (9), braid bars (7), and dunes (7).

The distance of a river delta avulsion node from the coast sets the scale over which deltas subaerially distribute sediment (Figure 5a). Field-scale delta avulsions cluster in the range $Bw = [5.3 \times 10^{-1}, 1.7 \times 10^{0}]$, a range of about half an order of magnitude (Figure 6), despite ranging in scale ($L$) over an order of magnitude. Laboratory avulsions span a similarly sized range of $Bw$ and substantially overlap the field-scale measurements ($Bw = [2.0 \times 10^{-1}, 9.2 \times 10^{-1}]$).

Channel mouth processes set the morphology and stratigraphic architecture for delta lobes (Figure 5b). Specific examples are the length scale that mouth bars can extend from a river mouth [Edmonds and Slingerland, 2007], the length of nonbifurcating channel extension [Rowland et al., 2009], the magnitude of shoreline perturbations in an experiment [Reitz and Jerolmack, 2012], or the growth of a prograding delta lobe [Shaw and Mohrig, 2014]. Field-scale measurements of distributary channels formed by channel mouth processes with length scales spanning 2 orders of magnitude exhibit $Bw = [7.4 \times 10^{0}, 3.0 \times 10^{1}]$, a range of about half an order of magnitude (Figure 6). Until recently, experimental studies had $Bw = [1.9 \times 10^{-1}, 3.3 \times 10^{0}]$, magnitudes that were considerably smaller than their field-scale counterparts. However, the recent experiments by Chatanantavet and Lamb [2014] that varied water discharge produced scours downstream of experimental river mouths with length scales that produced $Bw = 7.6 \times 10^{1}$.

River meanders grow from bank-attached point bars that evolve in concert with lateral river migration (Figure 5c). We thus investigate the scale of point bar bed forms using the meander wavelength [Williams, 1986]. Field studies with meander wavelengths $L$ ranging almost 3 orders of magnitude show $Bw = [7.1 \times 10^{0}, 1.8 \times 10^{1}]$, a range of less than half an order of magnitude (Figure 6). Experimental meanders to date have consistently smaller $Bw$ scale than their field counterparts, with $Bw = [4.0 \times 10^{-2}, 2.5 \times 10^{0}]$, about an order of magnitude lower than the field cases and ranging down into the space where bed forms are dominantly dispersive ($Bw < 10^{-1}$).

Braid bars are migrating (free) bars that separate the many channel courses in braided rivers (Figure 5d). Field-scale braid bars ranging in length over 2 orders of magnitude and show $Bw = [7.1 \times 10^{0}, 1.7 \times 10^{1}]$, a range of about half an order of magnitude. Laboratory-scale bars cover almost 2 orders of magnitude...
in $Bw$, and like meanders they have a lower, nonoverlapping $Bw$ range compared to the field studies ($Bw = [1.4 \times 10^{-1}, 3.6 \times 10^0]$).

Finally, dunes develop on the bottoms of alluvial channels but do not grow to the water surface as bars do (Figure 5b). The range of $Bw$ for field dunes spans about 3 orders of magnitude, $Bw = [4.9 \times 10^1, 2.6 \times 10^4]$ (Figure 6). Although the wide range of $Bw$ values found for field studies has not been found in laboratory studies, the experimental range of backwater values, $Bw = [7.4 \times 10^1, 2.8 \times 10^2]$, is completely overlapped by the $Bw$ range for field cases.

These data support hypothesis 2; distinct bed form types appear to have characteristic $Bw$ scales—particularly for field-scale bed forms. For river mouth processes, meanders, and braid bars, field-scale studies cluster within one half of an order of magnitude. Interestingly, the bed forms that require planform variations in flow direction—river mouths, meanders, and bars—all cluster in the same range near $Bw = 10^1$. Hypothesis 3, which bed forms easily produced in laboratory settings, has $Bw$ scales that closely match field-scale systems, received support from all bed form types except braid bars. Dunes and avulsions are easily reproduced in experiments and have similar $Bw$ ranges to natural systems. Meanders and channel mouth processes are difficult to recreate at laboratory scales, and laboratory experiments have significantly smaller $Bw$. However, braid bars, which are easily created in lab and field settings, also show a gap between field and experimental studies.

5. Interpretation and Discussion
5.1. Continuum Behavior of Bed Forms and $Bw$ Scaling

We have demonstrated that the backwater number captures a continuum between dispersive, mixed, and translative morphologic evolution of bed forms, confirming our first hypothesis. Our scaling analysis (section 2) shows that beds evolve dispersively when $Bw \ll 1$ and that beds evolve as kinematic waves when $Bw \gg 1$. Numerical results confirm the scaling analysis and further show that the transition between dominant dispersive and translative fluxes to be at $Bw \approx 0.5$ when $F_{Rb}$ is less than 0.6. Empirical study further corroborates hypothesis 1, finding that purely dispersive bed form evolution dominates for $Bw < 10^{-1}$, translation dominates for $Bw > 10^1$. However, bed form evolution with mixed dispersive and translative components occupies a large range of $Bw$ ([$7.5 \times 10^{-2}, 1.3 \times 10^1$]; Figure 5), indicating that the transition from dispersive to translative evolution is a gradual one lacking a precise $Bw$ threshold. These conclusions are drawn from an analysis that neglected the effects of grain mixtures, sources of resistance to flow other than sediment, and the threshold of motion despite their demonstrated roles in determining alluvial bed evolution [Bridge, 2009]. Even so, the backwater number still appears to be a first-order parameterization of the continuum of bed form behavior between translation and dispersion. It is possible that some of the neglected effects are embedded implicitly in the backwater number parameterization, but we see no compelling argument for their systematic control on the backwater number.

Previous work by Lisle et al. [1997, 2001] demonstrated that the Froude number controls whether sediment waves translate or disperse. Our numerical study supports this conclusion: for a given $Bw$, sediment flux associated with translation decreases with increasing $F_{Rb}$, particularly when $F_{Rb} > 0.6$. However, our numerical results also show that for a given $F_{Rb}$, the sediment flux associated with bed form translation also increases with increasing $Bw$ (Figures 2 and 3). Hence, $Bw$ can independently control the way a bed form evolves through differences in $H_p, S_{10}$, and $L$ and independently of $F_{Rb}$. Backwater scaling has the added benefit that all bed form scales in a single system can have different $Bw$ and thus evolve differently. For a system with a given $F_{Rb}$, some bed form scales might translate ($Bw \gg 1$), while others disperse ($Bw \ll 1$). For this reason, we consider $Bw$ a valuable new tool for predicting bed form behavior.

Backwater number scaling potentially provides an upper bound on bed form growth. In the absence of other limiting conditions such as bed material availability, nonequilibrium hydraulics, or a flow depth limit, growing bed forms might ultimately approach a condition where their length scale grows to near the backwater length and as a result begin to experience topographic dispersion in greater amounts, thus limiting their size. This limit could potentially be used as an upper bound for equilibrium bed form scale, and tool for interpreting bed forms when other formative conditions are unknown.

The empirical comparison of bed form behavior (Figure 4) could only be applied to one-dimensional bed forms because two-dimensional bed forms such as braid bars are more difficult to quantify on the translation-dispersion continuum. However, large braid bars on the Jamuna River in Bangladesh have been
observed to initiate with upstream erosion and downstream deposition and then to smear out due to bar-tail elongation [Ashworth et al., 2000]. This may be directly comparable to a shift from translation to dispersion. Future work may be able to quantify shifts on the dispersion-translation continuum and yield further insight into these general categories of bed form evolution.

5.2. Discrete Ranges of $B_w$ for Equilibrium Bed Forms

Our second hypothesis, which states that bed form types have characteristic $B_w$ scales, is supported by empirical analysis of a wide variety of bed forms (Tables 1 and 2 and Figure 6). Field studies of equilibrium bed forms showed clustering in about half of one order of magnitude for avulsions, channel mouth processes, meander bends, and braid bars. Dunes did not show a characteristic scale but had by far the largest $B_w$, exceeding 49 in all seven reported environments. The existence of a backwater scale is valuable, because it suggests that hydraulic conditions could be estimated from bed form scale or vice versa. We apply the assumption of distinct backwater scales to two problems in section 5.4.

Interestingly, river mouths, river bends, and braid bars all scaled in roughly the same range ($B_w = [7.1, 29]$). This prevents any distinction between these processes on the basis of $B_w$ alone. However, the tight clustering of these bed forms suggests a process-based control on their individual ranges. Bar and meander length-scales have previously been related to widths and depths of channels [Ikeda et al., 1981; Ikeda, 1984; Edmonds and Slingerland, 2007; Hundey and Ashmore, 2009; van Dijk et al., 2012]. Ikeda et al. [1981] successfully used linear stability analysis to determine the length scale of alluvial bends to be a function of flow depth and a dimensionless friction coefficient ($C_f$):

$$ L = \frac{2\pi H_0}{H_0 F_r^2} \cdot \left(1 - F_r^2\right) \cdot \frac{H_0}{S_f 0} \cdot \frac{S_f}{C_f} \cdot \frac{F_r^2}{C_f}.$$  \hspace{1cm} (17)

$C_f$ is related to shear stress through $\tau_b = \rho C_f U^2$. Applying this relation to (5), we see that when flow is uniform at the reach scale ($\frac{\partial U}{\partial x} = \frac{\partial H}{\partial x} = 0$), $\tau_b = \rho g H S_f = \rho C_f U^2$, and $S_f = C_f F_r^2$. We use this to rearrange equation 17 to solve for the backwater number for bars and meanders ($B_w_{bars}$):

$$ B_w_{bars} = \frac{0.24 \left(1 - F_r^2\right)}{F_r^2} \cdot \frac{H_0}{S_f 0} \cdot \frac{S_f}{C_f} \cdot \frac{F_r^2}{C_f}.$$  \hspace{1cm} (18)

Both sides of equation (18) contain $(1 - F_r^2)$ in the numerator, so the relation $\frac{S_f}{S_f 0} = \frac{0.24}{F_r^2}$ is also established. This shows that in the case of bars, the Froude number exerts a control on bar length scale relative to the hydrodynamic conditions. This trend is borne out for field and experimental braid bars and meanders in the data set (Figure 7), with $r^2 = 0.73$. In field settings, $F_r$ is commonly $0.1 - 0.3$, so equation (17) predicts $B_w_{bars}$ between 2.4 and 24. The compiled field data fall within this range. On the other hand, most experimental cases have $F_r$ between 0.3 and 1, which gives $B_w_{bars} < 2.4$. The compiled experimental data fall within this range. Equation (18) suggests that when $F_r > 0.57$, $B_w < 0.5$ and dispersive flux begins to dominate. According to one-dimensional numerical simulations (Figure 3c), bars would be in the diffusional regime at these conditions.
5.3. Comparison of Backwater Scales Between Laboratory and Field Systems

Is $Bw$ scaling an effective indicator of whether laboratory morphodynamics are similar to the field scale, consistent with our third hypothesis? The literature survey shows that dunes and delta avulsions have similar $Bw$ for the laboratory and the field scale despite a wide range of Froude numbers (Figure 6). Hence, $Bw$ scaling is an indicator of similarity between these experiments and field cases. In contrast, meander bends, braid bars, and river mouths have nonoverlapping $Bw$ ranges between experiments and field cases, with a single exception for river mouths. In section 5.2, we showed that $Bw$ was depressed for meander bends and braid bars due to heightened $Fr_0$ in experiments (Figure 7). However, experimental studies of avulsions cover the same range of $Bw$ as field studies despite significantly larger $Fr_0$. This is clear evidence that $Bw$ contains important information independent of the $Fr_0$ even in laboratory physical models.

Disparate scales of $Bw$ between laboratory and field braid bars is a surprising result. Braided planforms occur spontaneously in both laboratory and field settings, and similarity between experiments and the field has been demonstrated for sediment storage and release and the scour depth at confluences [Ashmore and Parker, 1983; Paola et al., 2009]. Even so, the break in $Bw$ between laboratory and field indicates that experimental bars should evolve more dispersively than their field-scale counterparts. Our one-dimensional model (Figures 2 and 3) shows that higher $Bw$ reduces downstream migration rate. While the downstream migration of braid bars is frequently noted in the field [Ashworth et al., 2000; Lunt and Bridge, 2004], we know of no experimental study where bars have migrated significant distances downstream. Furthermore, Bridge and Lunt [2006] note that experiments on braid bars have not yet resolved the smaller scales of sediment movement that are important in stratigraphic facies models. For these reasons, further comparative research between experimental and field-scale braid bars may heighten the understanding of braiding at both the field and laboratory scale.

Beyond the case of braiding, $Bw$ similarity is an accurate indicator of whether experimental and field systems behave in the same way. Meandering rivers are difficult to recreate in the lab, and $Bw$ is an order of magnitude smaller than in the field. Equation (18) suggests that reduced $Bw$ is the result of heightened $Fr$ (Figure 7). While the Froude dependence of bar formation is well established [Kleinhans et al., 2014], the importance of $Bw$ scaling provides a new explanation for morphodynamic difference between experimental and field scales: bar deposition and cut bank erosion are known to arise from spatial accelerations around a bar which scale with its length. If a bend is so long that the accelerations around it are small, then changes in sediment transport will also be minor, inhibiting both cut bank erosion and point bar deposition. This may explain the low sinuosities found in experimental meandering systems with $Bw \sim 1$.

Channel mouth processes are also difficult to compare between experiments and the field. Field measurements of distributary channels and channel mouths [Nittrouer et al., 2011a, 2011b; Shaw et al., 2013; Shaw and Mohrig, 2014] have shown that acceleration, erosion, and downstream translation of islands are commonplace. Laboratory studies have generally contrasting behavior, as river mouths $Bw < 3.5$ appear to be fully depositional [Edmonds et al., 2009; Hoyal and Sheets, 2009; Reitz et al., 2010; Reitz and Jerolmack, 2012]. A recent experiment by Chatanantavet and Lamb [2014] did produce erosion at the river mouth (albeit from unsteady flow) with conditions such that $Bw = 76$. These results suggest that accurate $Bw$ scaling could be the key to channel kinematics and river-mouth-scale stratigraphy that incorporates erosion as well as deposition.

5.4. Applying $Bw$ Theory

In field settings, backwater numbers for braid bars, meander bars, and river mouth processes show discrete, yet overlapping, ranges (Table 2 and Figure 6). If one assumes that this clustering is a general characteristic of these bed forms, $Bw$ scaling could be a useful tool in the analysis of planetary surfaces, stratigraphic outcrops, and for prediction necessary in landscape modeling. We demonstrate this with two example applications in stratigraphic and planetary surface analysis and discuss the use of $Bw$ scaling in models and as an upper bound of bed form size.

Our approach is applied to comprehensive observations of sand bodies in the Cretaceous Ferris Formation by Hajek et al. [2012]. They report an average height of bar clinoforms, $H = 0.59$ m, which we assume to be equal to average flow depth during bar formation. Combining this with a median grain size ($D_{50} \approx 0.4$ mm), a paleochannel slope can be estimated with the method of Trampush et al. [2014], which applies an empirical relationship between bankfull channel depth, median grain size, and slope, under the assumption of normal flow at the reach scale: $\log S = -2.08 + 0.254 \log D_{50} - 1.09 \log H_{bf}$ where $H_{bf}$ is bankfull flow depth and both $D_{50}$ and $H_{bf}$ are in meters. This results in a paleoslope estimate of $S = 2.0 \times 10^{-3}$. It is unclear whether the bars were constructed by braiding or meandering, so we assume the range of backwater scales for modern braid...
bars and meanders which have a combined range of $Bw$ between 7.1 and 18. Using equation (1) under the assumption of $Fr \ll 1$, we calculate the length scale of the Ferris Formation bars to be $L \approx [16, 41]$ m. This corresponds to the mean horizontal length of the outcropping sand bodies, $L = 19$ m (from Hajek et al. [2012], calculated from their Figure 4) which provides an initial confirmation of the method.

Backwater scaling can also provide context to bed forms on other planets. As long as $Fr_0 \ll 1$, then backwater scaling is theoretically independent of gravity, densities, or fluid viscosity (equation 1). Practically, important correlations between these parameters and bed form scale may exist, but the data to test this idea do not. In their absence, we apply backwater scaling with appropriate caution to alluvial stratigraphy of the Eberswalde delta, deposited during the Noachian Period in Holden Crater, Mars. The deposit records laterally accreting channel deposits interpreted to have formed by meanders with a characteristic wavelength of 1860 m [Moore et al., 2003]. The slope of the modern deposit was measured to be $6 \times 10^{-3}$ by the Mars Orbiter Laser Altimeter, MOLA, [Malin and Edgett, 2003]. Channel depths could not be measured directly but were assumed to be meter scale by Malin and Edgett [2003] and were estimated to be in the range $H=[3.6, 12.0]$ m by Jerolmack et al. [2004], dependent on the slope derived from MOLA. Applying these depth, slope, and length scales and assuming $Fr_0 \ll 1$ produces backwater numbers in the range $Bw=[0.1, 1.1]$. This range does not overlap with the range of backwater numbers for modern braid bars and meanders on Earth, $Bw \approx [7.1, 19]$. Note that a large Froude number due to Mars’ reduced gravity would further depress this $Bw$ estimate. If the MOLA-measured slope was indeed the depositional slope, then applying the observed range of backwater numbers for meanders and calculating the depths of the channels produces depths of $H=[79, 210]$ m which are unreasonable for channels measured to be approximately 100 m wide [Moore et al., 2003]. We find it more likely that the measured slope does not represent the characteristic bed slope during delta formation. Assuming that the range of depths estimated by Jerolmack et al. [2004] is valid, then backwater scaling from equation (1) produces bed slope estimates in the range of $S=[1.0 \times 10^{-4}, 9.1 \times 10^{-4}]$. The difference between formative and present-day surface slope could be due to differential erosion of the deposit or due to postdepositional tilting of approximately $0.29^\circ - 0.34^\circ$ to the east. Stratigraphic analysis of Lewis and Aharonson [2006] is consistent with the former explanation. This newly estimated formative slope may alter reconstructions of Martian paleoclimate [Irwin et al., 2014].

A third potential application of $Bw$ scaling could be in modeling of landscapes. The length scales of bars, meanders, and channel mouth processes could be estimated for restoration efforts on rivers and deltas. Landscapes generally have many bed form scales superimposed upon one another, and $Bw$ is an effective way to determine the behavior of each scale. It is even conceivable that large-scale landscape evolution models could use $Bw$ scaling to resolve bars, meanders, and channel mouths at the subgrid scale.

We see many potential applications for backwater theory. This study focused on fluvial-deltaic bed forms in unidirectional flow because flow depths and slopes are relatively easy to define. However, many bed form types were not included (e.g., knickpoints, deep-water bed forms, aeolian bed forms, and bed forms under oscillating flow) and could conceivably benefit from $Bw$ analysis. If the morphodynamics of a bed form depend on spatial accelerations of the flow field, its length scale and behavior might also depend in some way on a backwater number representative of these environments.

### 6. Summary and Conclusions

The backwater number, $Bw$, has not been extensively considered in alluvial bed form scaling to date. Here we rederived the sediment transport conditions captured by the backwater number from the shallow water equations and demonstrated their application to bed forms and sediment pulses. The salient physical insight of backwater number scaling is that spatial accelerations in the flow field are driven by changes in slope and/or changes in water depth. The transition between the dominance of changes in water depth to changes in slope occurs at the backwater length scale, $L_{bw}$. At longer bed form length scales, when $Bw \ll 1$, accelerations and stresses derive from slope changes, and alluvial forms result from dispersion-dominated transport. For bed form length scales shorter than $L_{bw}$ ($Bw \gg 1$), water depth changes are the primary factor determining accelerations and stress, and bed forms translate as kinematic waves. This creates the migrating wave-like behavior observed for equilibrium bed forms.

A 1-D numerical model of the shallow water equations coupled with sediment mass conservation (Appendix A) was used to explore the validity of backwater number scaling. This confirms that the proportion of sediment flux involved in bed form translation increases with increasing $Bw$ and is largely independent of
the Froude number for \( Fr_0 < 0.6 \). Conversely, the proportion of sediment flux responsible for topographic dispersion of sediment waves increases as \( Bw \) decreases. When \( Fr_0 \) increases above 0.6, dispersive flux increases independent of \( Bw \).

To empirically test the continuum of behavior (dispersion to translation) predicted by the backwater number and supported by the numerical model, we compiled a set of previously published data on one-dimensional sediment waves. This includes field and laboratory observations with backwater numbers spanning nearly 7 orders of magnitude [10^{-2}, 10^5]. Sediment pulses that exhibit purely dispersive behavior are found exclusively at backwater numbers less than 10^{-1}. Equilibrium bed forms that translate with mass conservation have only been observed with backwater numbers greater than 10^1. In the field and in laboratories, all systems with backwater numbers within approximately an order of magnitude of unity exhibit a mixed translative-dispersive behavior. This is consistent with the developed scaling theory and the numerical model.

Because of the strong connection between backwater scaling and behavior of sediment waves, we hypothesize that all equilibrium bed forms exist within characteristic ranges of backwater number. We find this to be true: field-scale bed forms clustered within an order of magnitude or less for each bed form type. Further, the ranges of \( Bw \) for field-scale dunes and delta avulsions overlap the ranges of their experimental analogues. In contrast, ranges of \( Bw \) for experimental meanders and braid bars and most experimental river mouths do not overlap the corresponding ranges from field settings. The ease of achieving similarity of \( Bw \) between the laboratory and field settings for avulsions and dunes corroborates why they have been effectively reproduced. The disconnect between field and laboratory backwater numbers for bars and meanders appears to be explained by the high Froude numbers in experimental settings. This dissimilarity may shed light on differences in evolution for these bed forms between field and laboratory settings.

Backwater number scaling captures a continuum of bed form behavior. Further, distinct, equilibrium alluvial forms have characteristic ranges of \( Bw \). Finally, the simplicity of \( Bw \) scaling allows it to be applied in a wide variety of settings. This universal approach is a step forward in understanding sedimentary systems as a continuum of bed forms and behaviors.

Appendix A: Numerical Model Description

We simulate the one-dimensional evolution of alluvial bed forms captured by the shallow water equations, sediment-mass conservation, and bed load flux to illustrate their dependence on the backwater number, \( Bw \) (Figures 2 and 3). The initial conditions for each run included a prescribed bed configuration \( h_0 = f(x) \), a downstream water depth \( H_{end} \), and a width-averaged water discharge \( q_w \) boundary condition. The model iterated over the following cycle: first, the water depth along the domain was resolved by combining equations (5) and (6) and approximating shear stress as \( \tau_x = \rho \frac{C_f}{U^2} \) to produce the well-known backwater equation

\[
\frac{\partial H}{\partial x} = \frac{-\frac{2}{3} U - C_f F r^2}{1 - F r^2},
\]  

(A1)

\( C_f \) was set at 0.04, consistent with many sand-bed rivers [García, 2007]. Next, water depth, downstream velocity, and shear stress were computed over the domain by integrating up from the downstream boundary where \( H = H_{end} \). Once shear stress was known across the domain, sediment flux was calculated using equation (3). We used \( \alpha = 5.0 \times 10^{-6} \text{m}^2/\text{s} \) and \( \beta = 2.5 \) which is equivalent to 100 \( \mu \text{m} \) sand transported by the Engelund and Hansen [1967] transport formula. The sediment flux at the downstream and upstream ends of the domain were set equal to simulate a periodic domain. Finally, the bed evolved through equation (2) over a small time interval \( \Delta t \) assuming a bed concentration of \( e = 0.65 \). This cycle iterated with the newly updated topography to the first step above.

The boundary conditions were computed for desired values of \( Bw \) and \( Fr_0 \). The average bed slope was set to be the slope which produced uniform flow \( (H(x) = H_{end}) \) in the absence of bed perturbations in equation (A1), i.e., \( S = C_f F r_0^2 \). The downstream water depth condition could then be calculated from (1) as \( H_{end} = Bw S L / (1 - F r_0^2) \). Finally, width-averaged water discharge \( q_w = \rho H U \) was set so as to impose uniform flow over a flat bed of slope \( S \). This was done by modifying equation (A1) so that \( \partial H / \partial x = 0 \) and \( -\partial \eta / \partial x = S \). Rewriting the Froude number so that \( Fr = q_w / \sqrt{g H^3} \), we solve for \( q_w \):

\[
q_w = \sqrt{\frac{g H^3_{end} S}{C_f}},
\]  

(A2)
Each run simulated four sinusoidal bed forms of wavelength $L = 100\,\text{m}$ held constant for all simulations. The initial amplitude $h_d$ was set as $h_d = 0.1L/(2\pi)$ to ensure that although there were bed perturbations, the bed was always sloping in the downstream direction. Many dunes have adverse bed slopes, but adverse bed slopes for $B_w < 1$ produce pooling upstream of bed forms which adds complexity to the bed evolution that we did not investigate. The $h_d$ criterion ensures that a wide variety of $B_w$ can be modeled and directly compared simply by varying water depth in the system. The initial bed elevation for $x = [0\,\text{m}, 400\,\text{m}]$ was

$$\eta(x) = -Sx - \frac{h_d}{2}\cos\left(\frac{2\pi x}{L}\right).$$  \hspace{1cm} (A3)

The widely varying water depths between runs caused large differences in sediment transport and bed evolution rates. To compare between runs, we normalized the run time by the time needed to transport half the sediment in one bed form. Bed evolution is set by spatial differences in sediment transport, so effective sediment flux ($\Delta q_s$) was set as the difference between the maximum and minimum sediment fluxes at the beginning of each simulation ($\Delta q_s = \max q_s - \min q_s$). The amount of sediment in each initial bed form is $0.5h_dL$. Thus, the total run time of each run was set to be $0.5h_dL/\Delta q_s$. This was broken up into 1000 time steps, so $\Delta t = 0.001h_dL/\Delta q_s$.

**Notation**

- $B_w$: backwater number, ($-$), equation (1).
- $C_f$: friction coefficient ($-$).
- $Fr$: Froude number ($-$).
- $Fr_0$: characteristic Froude number ($-$).
- $g$: acceleration due to gravity ($\text{m}\,\text{s}^{-2}$).
- $H$: flow depth ($\text{m}$).
- $H_0$: characteristic flow depth ($\text{m}$).
- $H_{end}$: flow depth at downstream boundary of model ($\text{m}$).
- $h_d$: bed form amplitude ($\text{m}$).
- $L$: bed form length scale ($\text{m}$).
- $L_{bw}$: backwater length scale ($\text{m}$).
- $q_s$: width-averaged sediment flux ($\text{m}^2\,\text{s}^{-1}$).
- $\Delta q_s$: sediment flux associated with bed evolution ($\text{m}^2\,\text{s}^{-1}$).
- $R$: correlation coefficient ($-$).
- $S$: characteristic bed slope ($-$).
- $S_f$: energy slope ($-$), equation (9).
- $S_{f0}$: characteristic energy slope ($-$).
- $t$: time ($\text{s}$).
- $U$: depth-averaged flow velocity in the downstream direction ($\text{m}\,\text{s}^{-1}$).
- $U_0$: characteristic flow velocity ($\text{m}\,\text{s}^{-1}$).
- $V$: depth-averaged flow velocity in the cross-stream direction ($\text{m}\,\text{s}^{-1}$).
- $V_c$: bed form celerity ($\text{m}\,\text{s}^{-1}$).
- $z$: distance above the bed ($\text{m}$).
- $z_0$: roughness length ($\text{m}$).
- $\alpha$: coefficient.
- $\beta$: coefficient.
- $\epsilon$: concentration of sediment in bed (1).
- $\eta$: bed elevation ($\text{m}$).
- $\tau_b$: basal shear stress ($\text{km}\,\text{m}^{-1}\,\text{s}^{-1}$).
- $\tau_{cr}$: critical shear stress for sediment movement ($\text{km}\,\text{m}^{-1}\,\text{s}^{-1}$).

**References**


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